# TAQSIMOT FUNKSIYA VA UNING XOSSALARI

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**Annotatsiya:** Simmetrik fazolarni ta’rifini kiritish uchun bizga taqsimot funksiya va teng o‘lchovlilik tushunchalari zarur bo‘ladi. Ushbu ishda taqsimot funksiya va teng o‘lchovlilik tushunchalari ta’riflanadi. Taqsimot funksiyaning xossalarini sanab o‘tiadi.

**Kalit so‘zlar.** Taqsimot funksiya, teng o‘lchovli, $\sigma$–algebra, o‘ngdan uzluksiz funksiya, Lebeg ma’nosidagi o‘lchov.

Faraz qilaylik, $R^+ = [0,\infty)$ va $m$ - o‘lchov Lebegning $R^+$ yarim to‘g‘ri chiziqdagi odatiy o‘lchovi bo‘lsin. Bu shuni anglatadiki, $m$ - o‘lchovning $\mathrm{B} = \mathrm{B}(R^+)$ $\sigma$–algebradagi qisqartma (sujeniya) si $m|_{\mathrm{B}}$ – $R^+$ dagi yagona borel o‘lchovi bo‘lib,

$m([0,x]) = x,\ x \in R^+$.

$m$ - o‘lchovning o‘zi esa, bu $m|_{\mathrm{B}}$ o‘lchovning $R^+$ ning Lebeg ma’nosida o‘lchovli bo‘lgan qism to‘plamlaridan iborat $\sigma$–algebra $F = F(R^+)$ gacha davomi bo‘lib hisoblanadi.

Har bir nomanfiy o‘lchovli funksiya $f : R^+ \to R^+$ ga

$$\eta_f(y) := m\{f > y\}, y \in R^+, \quad (1)$$

ni mos qo‘yamiz, bunda

$$\{f > y\} := \{x \in R^+ : f(x) > y\},\ y \in R^+$$

– $f$ funksiyaning yuqori lebeg to‘plami.

**1-ta’rif.** (1) tenglik yordamida aniqlangan $\eta_f(y):[0,+\infty)\to[0,+\infty]$ funksiya $f$ funksiya uchun (yuqori) taqsimot funksiya deb ataladi.

Taqsimot funksiyaning xossalarini sanab o‘tamiz.

**1-tasdiq.** Yuqori taqsimot funksiya $\eta_f$ – kamayuvchi funksiya.

Isbot. Teoremaning isboti

$$y_1<y_2 \Rightarrow \{f>y_1\}\supseteq\{f>y_2\}\Rightarrow \eta_f(y_1)\geq\eta_f(y_2)$$

munosabatdan kelib chiqadi.

**2-tasdiq.** Yuqori taqsimot funksiya $\eta_f$ – o‘ngdan uzluksiz funksiya.

Isbot. Teoremaning isboti

$$y_1\downarrow y_2 \Rightarrow \{f>y_1\}\uparrow\{f>y_2\}\Rightarrow \eta_f(y_1)\uparrow\eta_f(y_2)$$

munosabatdan kelib chiqadi.

Shuni ta’kidlab o‘tamizki, o‘ngdan uzluksizlik (1) dagi qat’iy tengsizlik “ > ” ni tanlash orqali aniqlanadi. Ushbu $y\to\eta_f(y)$ funksiyadan farqli ravishda, $\eta\to m\{f\geq y\}, y\in R^+$ funksiya chapdan uzluksizdir.

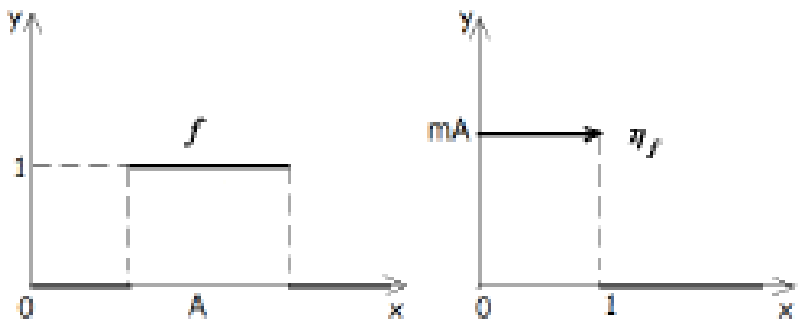
**1-misol.**

1. Aytaylik, $f=1_A$ funksiya $A\in F_m$ to‘plamning indikator (xarakteristik) funksiyasi bo‘lsin:

$$1_A(x)=\begin{cases}1, \text{агар } x\in A \text{ бўлса},\\ 0, \text{агар } x\notin A \text{ бўлса}.\end{cases}$$

U holda $\eta_f=mA\cdot 1_{[0,1)}$, ya’ni

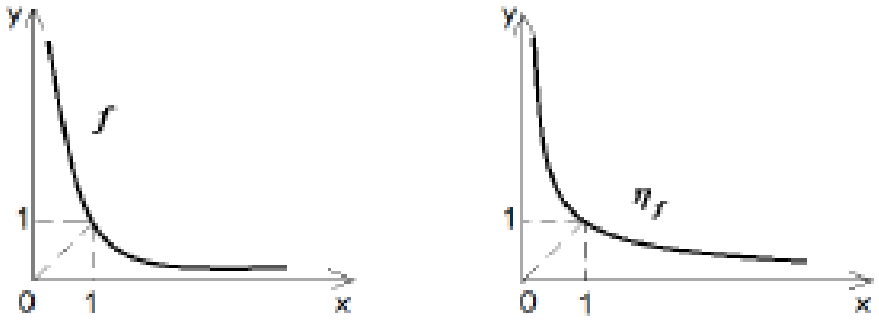
$$\eta_f(x)=\begin{cases}mA, \text{агар } 0\leq x<1 \text{бўлса},\\ 0, \quad \text{агар } x\geq 1 \text{бўлса}.\end{cases}$$




1-rasm.

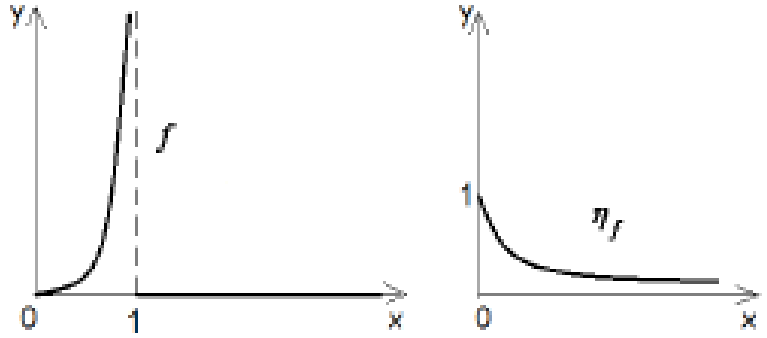
Yana shuni ta’kidlash kerakki, agar $mA=\infty$ bo‘lsa, u holda $\eta_f(x)=+\infty$, $0\leq x<1$ bo‘ladi.

2. $f(x)=\frac{1}{x^2}$ bo‘lsin. U holda $\eta_f(x)=\frac{1}{\sqrt{x}}$.




2-rasm.

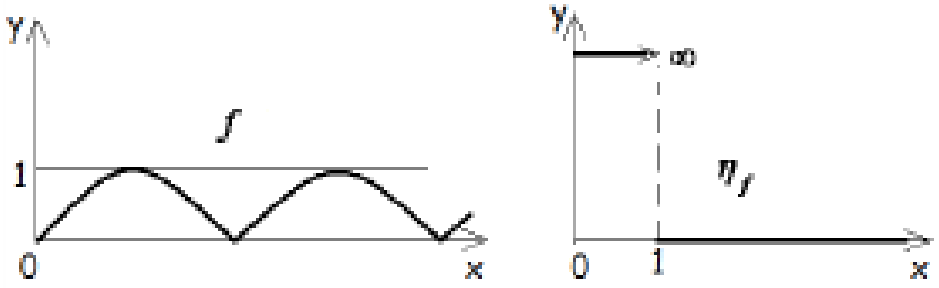
3. $f(x)=\frac{x}{1-x}\cdot 1_{(0,1)}(x)$ bo‘lsin. U holda $\eta_f(x)=\frac{1}{x+1}$.




3-rasm.

4. $f(x)=|\sin x|$ bo‘lsin. U holda

$$\eta_f(x)=\begin{cases}\infty, \text{агар } 0\le x<1 \text{бўлса},\\ 0, \quad \text{агар } 1\le x \text{бўлса}.\end{cases}$$




4-rasm.

**2–ta’rif.** Ikkita nomanfiy $f$ va $g$ funksiyalar uchun, ixtiyoriy $y\in[0,\infty)$ larda $\eta_f=\eta_g$ munosabat, ya’ni

$$m\{f>y\}=m\{g>y\}$$

tenglik o‘rinli bo‘lsa, $f$ va $g$ funksiyalar teng o‘lchovli deb ataladi.

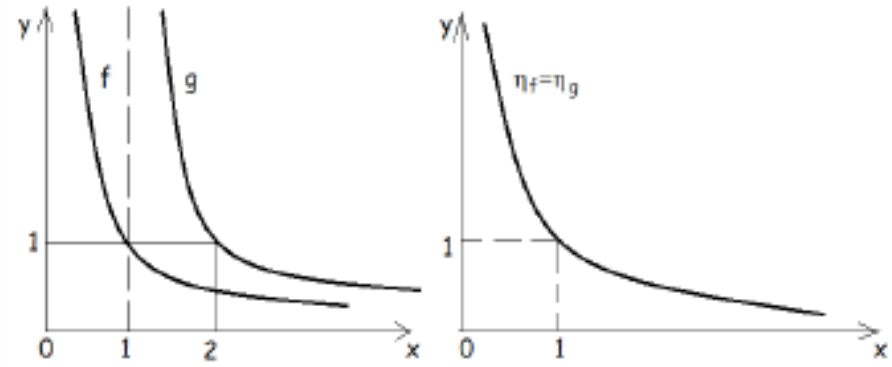
**2-misol.**

1. $A, B \in F_m$ bo‘lsin. Ikkita xarakteristik funksiyalar $1_A$ va $1_B$ $mA = mB$ bo‘lganda, faqat shu holda teng o‘lchovli funksiyalar bo‘ladi.

2. Ushbu

$$f(x) = \frac{1}{x^2} \text{ va } g(x) = \frac{1}{(x-1)^2} \cdot 1_{[1,+\infty)}(x)$$

funksiyalar teng o‘lchovli funksiyalardir, chunki, $\eta_f(x) = \eta_g(x) = \frac{1}{\sqrt{x}}$.




5-rasm.

**Foydalanilgan adabiyotlar ro‘yxati:**

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